Progic 2009: 4th Workshop on Combining Probability and Logic

17–19 September 2009, University of Groningen, The Netherlands

SAINT LOUIS UNIVERSITY

Department of Philosophy
Saint Louis University

❖ **JOHN R. WELCH**

http://spain.slu.edu/fac/jwelch7.html

❖ DECISION THEORY AND RATIONAL CHOICE

A COMPLAINT ABOUT DECISION THEORY

"Decision theory tends to work best for trivial decisions, such as when you are in a casino, trying to decide whether to play craps or roulette. For life's biggest decisions, such as whether to get married or have children, it is pretty much useless."

William B. Irvine, *On Desire: Why We Want What We Want* (Oxford: Oxford University Press, **2006**), p. 112.

Why is decision theory "pretty much useless" for "life's biggest decisions"?

NUMERIC POVERTY

For life's biggest—and many smaller—decisions, we rarely have sharp numeric values for probabilities of states and utilities of outcomes.

One attempt to solve the problem

- Drop back from point values to intervals.
- Express probabilities and utilities as numeric intervals, e.g., ⅓ – ½.

- Henry E. Kyburg, Jr., "*Tyche and Athena*" Synthese 40 (**1979**), 415–438.
- Mark Kaplan, *Decision Theory as Philosophy* (Cambridge: Cambridge University Press, **1996**).

A new attempt to solve the problem

- Drop back further still from intervals to comparisons.
- Express probabilities and utilities in quantitative but non-numeric terms, e.g., >, =,<.
- Develop a version of **comparative decision theory**.

WORKING ASSUMPTIONS

1) Propositional basics

- Decision-theoretic acts, states, and outcomes can be thought of as propositions.

  Richard C. Jeffrey, *The Logic of Decision*, 2nd ed. (Chicago and London: University of Chicago Press, **1983**), pp. 82–85.

- Acts, states, and outcomes are assumed to be finite.

WORKING ASSUMPTIONS

2) Acts

- Although comparative decision theory is not limited to a specific field of application, for the sake of illustration, the acts to be considered here are mental acts of choosing a theory.

WORKING ASSUMPTIONS

3) States

- Generalize probability of states as plausibility of states.

- Plausibility values include two nonnumeric limits, ⊥ and ⊤, such that, for any plausibility value x, $\bot \leq x \leq \top$.

  - Nir Friedman and Joseph Y. Halpern, "Plausibility Measures: A User's Guide," *Proceedings of the Eleventh Conference on Uncertainty in Artificial Intelligence (UAI '**95**)*, 175–184.

  - Joseph Y. Halpern, *Reasoning about Uncertainty* (Cambridge, MA: MIT Press, **2003**), pp. 50–51.

WORKING ASSUMPTIONS

4) Outcomes

- The relevant outcome in <u>cognitive</u> theory choice is information.

- Information is reduction of uncertainty about possible states of the world.

WORKING ASSUMPTIONS

5) Decision rule

- "*Plausibilize*" the decision rule.
- Instead of calculating the expected utility *E* of an action *a*:

$$E_{a,e} = \sum_{i=1}^{n} \upsilon(o_i)\mu(s_i, e),$$

calculate its plausibilistic expectation *PE*:

$$PE_{a,e} = \sum_{i=1}^{n} \upsilon(o_i)\pi(s_i, e).$$

- Instead of maximizing expected utility, maximize plausibilistic expectation.

Origin of this decision rule

- Chu and Halpern define a notion of generalized expected utility that employs plausibility in place of probability.

  Francis C. Chu and Joseph Y. Halpern, "Great Expectations. Part I: On the Customizability of Generalized Expected Utility," *Theory and Decision* 64 (**2008**), 10.

- They also derive a "universal" decision rule that returns the same ordinal rankings as any decision rule that satisfies the trivial condition of weakly respecting utility: act preferences track outcome utilities for all constant acts.

  Francis C. Chu and Joseph Y. Halpern, "Great Expectations. Part II: Generalized Expected Utility as a Universal Decision Rule," *Artificial Intelligence* 159 (**2004**), 211–212.

- The decision rule for plausibilistic expectation is a special case of this "universal" decision rule.

WORKING ASSUMPTIONS

6) Relative disutility

- What is the disutility of a mistaken cognitive choice?
- "If *h* is true, the utility of his decision is the valid information he has gained…. If *h* is false, it is natural to say that his disutility or loss is measured by the information he lost because of his wrong choice…."

  Jaakko Hintikka and Juhani Pietarinen, "Semantic Information and Inductive Logic," in *Aspects of Inductive Logic*, ed. Jaakko Hintikka and Patrick Suppes (Amsterdam: North-Holland, **1966**), pp. 107–108.
- Choice of *h* → $u(i_h)$ or $-u(i_{-h})$.
- To be used **ONLY** when reliable numeric utilities are not available.

WORKING ASSUMPTIONS

7) Relations between theories

- Plausibility primitive: $\prec$
- Infraplausibility: $\pi(s_1,e) < \pi(s_2,e)$
- Supraplausibility: $\pi(s_1,e) > \pi(s_2,e)$
- Equiplausibility: $\pi(s_1,e) = \pi(s_2,e)$
- Noncomparability: $\pi(s_1,e) \mid \pi(s_2,e)$
- Structurally comparable relations can be defined for utility and plausibilistic expectation.

BINARY THEORY CHOICE

CASE	PLAUSIBILITY	UTILITY
1	$\pi(s_1,e) < \pi(s_2,e)$	$\upsilon(o_1) < \upsilon(o_2)$
2	$<$	$>$
3	$<$	$=$
4	$<$	$\vert$
5	$>$	$<$
6	$>$	$>$
7	$>$	$=$
8	$>$	$\vert$
9	$=$	$<$
10	$=$	$>$
11	$=$	$=$
12	$=$	$\vert$
13	$\vert$	$<$
14	$\vert$	$>$
15	$\vert$	$=$
16	$\vert$	$\vert$

CASE 1

- Where $P > p$ and $U > u$, the plausibilistic expectation of t_1 is

$$PE_1 = up - UP,$$

and the plausibilistic expectation of t_2 is

$$PE_2 = UP - up.$$

- Since PE_1 is negative while PE_2 is positive, the comparative decision-theoretic choice would be t_2.

Results for the binary case

<table>
<tr><th>CASE</th><th>PLAUSIBILITY</th><th>UTILITY</th><th>RESOLUTION</th></tr>
<tr><td>1</td><td><</td><td><</td><td>t_2</td></tr>
<tr><td>2</td><td><</td><td>></td><td>no decision</td></tr>
<tr><td>3</td><td><</td><td>=</td><td>t_2</td></tr>
<tr><td>4</td><td><</td><td>|</td><td>t_2</td></tr>
<tr><td>5</td><td>></td><td><</td><td>no decision</td></tr>
<tr><td>6</td><td>></td><td>></td><td>t_1</td></tr>
<tr><td>7</td><td>></td><td>=</td><td>t_1</td></tr>
<tr><td>8</td><td>></td><td>|</td><td>t_1</td></tr>
<tr><td>9</td><td>=</td><td><</td><td>t_2</td></tr>
<tr><td>10</td><td>=</td><td>></td><td>t_1</td></tr>
<tr><td>11</td><td>=</td><td>=</td><td>t_1 or t_2</td></tr>
<tr><td>12</td><td>=</td><td>|</td><td>t_1 or t_2</td></tr>
<tr><td>13</td><td>|</td><td><</td><td>t_2</td></tr>
<tr><td>14</td><td>|</td><td>></td><td>t_1</td></tr>
<tr><td>15</td><td>|</td><td>=</td><td>t_1 or t_2</td></tr>
<tr><td>16</td><td>|</td><td>|</td><td>no decision</td></tr>
</table>

A note on cases 11, 12, and 15

- In general, binary choice presents 4 possible choices: option 1, option 2, both options, neither option.
- The result in these three cases is ***t_1 or t_2***. Hence 2 of these 4 options have been eliminated: neither and both (since the theories are rivals).
- Standard numeric decision theory yields comparable, non-unique results: numeric ties.

THE FINITE GENERAL CASE

- The number of serious candidate theories under consideration by an agent at any given moment appears to be always, or almost always, finite and small.
- Preference among theories is a transitive relation.
- Hence successive binary comparisons can be performed for the set of serious candidate theories.

CONCLUSIONS

- Comparative decision theory determines binary theory choice in thirteen of sixteen cases. Standard numeric forms of decision theory determine fifteen of the sixteen cases.

- But comparative decision theory can very frequently be applied where standard numeric forms of decision theory cannot.